Binary Notes pg 1

binary digit

AP CS Principles Binary Supplement

"A bit, or binary digit, is the smallest unit of information processed by a computer and consists of a single zero or one. A byte consists of 8 adjacent bits."

"The transistor, the basic building block of the CPU and RAM, is a simple device that can in one of two states – ON, conducting electricity, or OFF, not conducting electricity. All the information in a computer – programs and data – is expressed in terms of these ONs and OFFs, 1s and 0s, as they are more conveniently called."

2	2^1
4	2^2
8	2^3
16	2^4
32	2^5
64	2^6
128	2^7
256	2^8

n bits can be used to represent 2^n different values. (*** see extension notes.) Therefore 8 bits represent 2^8 or 256 values and 16 bits represent 2^{16} or 65536 values.

Java uses 4 bytes to represent an int and therefore can store 2^{32} different values: but these are split between negatives & positives.

A double is 8 bytes long. 1 bit is used for the sign, 52 bits for the coefficient, and 11 bits for the exponent.

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In base 10, the number 5403 can be expressed as: $5 * 10^3 + 4 * 10^2 + 0 * 10^1 + 3 * 10^0$ (thousands, hundreds, tens, ones)

NOTE: 5000+ 400 + 0 + 3 = 5403. We sum the value of each position in base 10. (and other bases as well!)

The right-most digit tells you how many ones you have; the next digit tells you how many tens you have and so on. The same concept can be applied to numbers expressed in other bases. For example, convert the following values to base 10.

374_8 is equal to $0 * 8^3 + 3 * 8^2 + 7 * 8^1 + 4 * 8^0 =$ 444 in base 10 (6*64 + 56 + 4, 384 + 60 =)

1101_2 is equal to $1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 =$ 13 in base 10 (8 + 4 + 1)

53_9 is equal to $0 * 9^3 + 0 * 9^2 + 5 * 9^1 + 3 * 9^0 =$ 48 in base 10 (45 + 3)

on board:

thousands place	hundreds place	tens place	ones place
10^3	10^2	10^1	10^0

how many of each of these? practice

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Backtrack on notes at each math concept.

n bits can be used to represent 2^n values? Hmmm lets see. . .
If each place can be only 2 different things, a one or a zero

1 place:	1	0	2 values	2^1
2 places:	1 1 0 1	1 0 0 0	4 values	2^2
3 places:	1 1 1 1 1 0 0 1 1 0 1 0	1 0 1 1 0 0 0 0 1 0 0 0	8 values	2^3
4 places:			16 values	2^4
n places:				2^n

Also look at base 10 numbers:

what digits do we have for base 10?
0 thru 9 inclusive (meaning includes both ends)

what digits do we have for base 2? ?
0 thru 1 inclusive (meaning includes both ends)

soooo what digits do we have for base 3?
? 0 thru 2 inclusive (meaning includes both ends)

what digits do we have for base 4?
? 0 thru 3 inclusive (meaning includes both ends)

what digits do we have for base n?
? 0 thru n-1 inclusive (meaning includes both ends)

extension:
how many values can be represented by base 3 with 2 places? 9 0, 1, 2 $3^2 = 9$
what is the range of values? 0 - 8

how many values can be represented by base 3 with 3 places? 27 $3^3 = 27$
what is the range of values? 0 - 26 → proof: 2 2 2

3^2 3^1 3^0

$9*2 + 3*2 + 2*1$

$18 + 6 + 2$

26

how many values can be represented by base 3 with n places? 3^n
what is the range of values? 0 to $3^n - 1$

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☆'s by what you must Know.... Binary I

AP Computer Science
Unit 2. Exercises

A common standard is 24-bit color where 8 bits are used to represent the amount of red light, 8 bits for green light, and 8 bits for blue light. It is the combination of these three colors that generates every other color you see on a monitor.

go back to binary notes!

1. How many different levels of red can be represented using 8 bits? 2^8 or 256
2. How many different colors can be represented using 24 bits? 2^{24}

2^{16} times more

Y2K refers to a computer problem from the previous century. In the early days of computers, memory (whether for storage or running a program) was very expensive. So, when recording dates, only the last two digits were saved. For example, the year 1985 would be saved as 85. This worked well enough for most programs at first but with the approach of the year 2000, it needed to be fixed; otherwise the year 2000 would be indistinguishable from the year 1900 (as both would be saved as 00).

$2^6 = 64$ $2^7 = 128$

3. How many bits are necessary to save a number from 0 to 99? 7 bits

4. How many bits are necessary to save a number from 0 to 9999? 14 bits

$2^{10} = 1024$ $2^{12} = 4096$ $2^{11} = 2048$ $2^{13} = 8192$ $2^{14} = 16,384$

5. An Arduino Uno microcontroller can detect a voltage between 0 and 5 volts. How many bits does it use to represent the voltage if 1023 indicates 5 volts and 0 indicates 0 volts? 10 bit bc $2^{10} = 1024$

in other words if 1023 is largest number....

Translate each binary number into a base ten number.

6) 1010_2 — 10	7) $0010\ 0001_2$ — 33_{10}
8) 1111_2 — 15_{10}	9) $1000\ 0000_2$ — 128_{10}
10) $1111\ 1111_2$ — 255_{10}	11) $1000\ 1000_2$ — 136_{10}

Translate each hexadecimal number into a base ten number

12) 31_{16} — 49_{10}	13) $2F_{16}$ — 47_{10}
14) $5D_{16}$ — 93_{10}	15) $A5_{16}$ — 165_{10}
16) 59_{16} — 89_{10}	17) 12_{16} — 18_{10}

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HEX

Numbers written to base 16 are hexadecimal numbers and the following letters are used to represent different values: A = 10, B = 11, C = 12, D = 13, E, = 14, F = 15   32 + 15

$2F_{16}$ is equal to ____ * 16^3 + ___ * 16^2 + 2 * 16^1 + F * 16^0 = 47 in base 10

you expand the hex number below to change it into base 10

$A4_{16}$ is equal to ____ * ____ + A * 16^1 + 4 * 16^0 = 164 in base 10

160   4

Here is one way to convert a number from base 10 to another base. To demonstrate this approach, let's say that we want to convert 70 base 10 to base 4. NOTE: this is more difficult.
First, construct a table with enough columns so that the leftmost column represents a number larger than the number you are converting. Beneath each base ^ exponent, write the equivalent value in non-exponential form. Finally, in the bottom row write out how many of each you need to represent your target. In the table below, you need one 64, one 4, and two ones to represent the number 70.

Exponents	4^4	4^3	4^2	4^1	4^0
Value base 10	256	64	16	4	1
To represent 70 you will need		1	0	1	2

Therefore, $70_{10} = 1012_4$□ 70/64 = 1 70 – 64 = 6 6/16 = 0 6/4 = 1 6 – 4 = 2

To convert 440 base 10 to hex ...

Therefore, 440_{10} =□

Exponents	16^3	16^2	16^1	16^0
Value base 10	4096	256	16	1
To represent 440 you will need	0	1	11	8

Therefore, 440_{10} = 1B8 $_{16}$□

440 / 4096 = 0 440/256 = 1 440 – 256 = 184
184/16 = 11 (B) 184 – (16)(11) = 8

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1437_{16}

To convert 5175 base 10 to hexadecimal

Exponents	16^3	16^2	16^1	16^0
Value hexadecimal	4096	256	16	1
To represent 5175 you will need	1	4	3	7

5175 − 4096 = 1079   1079 − 1024 = 55   55 − 48 = 7

256 * 4 = 1024   16 * 3 = 48

Later we will translate numbers from base 2 to hex and hex to base 2. We will translate numbers from base 4 to base 8 and even add numbers in different bases. We will practice our base system understanding over the next several days and continue to revisit this throughout the year. It is a good skill for a computer scientist.

$42_5 \rightarrow \underline{22}_{10}$ $4 * 5^1 + 2 * 1$

$17_{10} \rightarrow \underline{10001}_2$

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
1	0	0	0	1

$53_{10} \rightarrow \underline{104}_7$

7^2	7^1	7^0
49	7	1
1	0	4

53 − 49 = 4

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Translate each number into a base 10 number. If a number is incorrectly represented, and therefore cannot be translated, write INVALID.

18)	22_3	$2*3^1 + 2*1 = 6+2 = 8$	19)	80_7	$8*7^1 + 0*1 = 56$
20)	31_5	$3*5^1 + 1*1 = 16$	21)	123_4	$1*4^2 + 2*4^1 + 3*4^0$ $16 + 8 + 3 = 27$
22)	272_8	$2*8^2 + 7*8^1 + 2*1$ $128 + 56 + 2 = 186$	23)	55_6	$5*6^1 + 5*1 = 35$

Translate each base ten integer into a binary number.

24)	67	1000011_2	25)	100	1100100_2
26)	8	1000_2	27)	12	1100_2
28)	50	110010_2	29)	88	1011000_2

Translate each base ten number into a hexadecimal number.

30)	30	$1E_{16}$	31)	123	$7B_{16}$
32)	8	8_{16}	33)	64	40_{16}
34)	54	36_{16}	35)	75	$4B_{16}$

36) Convert this integer from base 10 to base 5

24 44_5

exp	5^2	5^1	5^0
value	25	5	1
	0	4	4

37) Convert this integer from base 10 to base 9

18 20_9

exp	9^1	9^0
value	9	1
	2	0

38) Convert this integer from base 10 to base 7

60 114_7

exp	7^2	7^1	7^0
value	49	7	1
	1	1	4

60 − 49 = 11

39) Convert this integer from base 10 to base 3

23 212_3

exp	3^3	3^2	3^1	3^0
value	27	9	3	1
	0	2	1	2

23 − 18 = 5

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Name ______________

Switching between Base 2 and any other base that is a multiple of 2

Lets start with the traditional base 2 to base 16 (hexadecimal)

Using base 2 (only 1s and 0s) how many digits or places would you need to represent a hex digit? Well hex numbers are base 16 and therefore can represent the digits from 0 through 15 (remember that 10 is A, 11 is B, etc). To represent 0 through 15 using base 2 or binary you need 4 places:

0 0 0 0 = 0 $\quad 0 * 2^3 + 0 * 2^2 + 0 * 2^1 + 0 * 2^0$

1 1 1 1 = 15 $\quad 1 * 2^3 + 1 * 2^2 + 1 * 2^1 + 1 * 2^0$ $\quad$ 8 + 4 + 2 + 1 = 15

What this means is that for every 4 digits in base 2 we can group them together to make one hexadecimal digit! Check it out below:

0010/1010/0011 == 2/10/3 == $2A3_{16}$ (10 is A in hex)

Proof, convert each number to decimal:

Binary 0010 1010 0011 = $1 * 2^9 + 1 * 2^7 + 1 * 2^5 + 1 * 2^1 + 1 * 2^0$
= 512 + 128 + 32 + 2 + 1 = 675

Hex 0010 1010 0011 = $2 * 16^2 + A * 16^1 + 3 * 16^0$
= 512 + 160 + 3 = 675

Now you try converting between base 2 and hexadecimal or base 16:

$0110\ 1110\ 1010_2$ == 6EA $_{hex}$

$10\ 1111\ 0110\ 0010_2$ == 2F62 $_{hex}$

Try the backside for more practice:

10 A
11 B
12 C
13 D
14 E
15 F

BASE COMPARISON using quick change method

	Binary	> < or =	Hexadecimal
	11011100	>	75
	10101011	=	AB
2D	101101	<	2F
	1011000	=	58
C8	11001000	>	B2
~~AE~~	01011110	>	5E
B1	10110001	<	B2
A	1010	<	14
64	1100100	>	62
3E8	1111101000	>	3DD
3DD	001111011101	>	3D3
28E	1010001110	=	28E
1F6	111110100	<	226
58	1011000	<	66
A7	10100111	=	A7
2	10	<	10
11	00010001	=	11

CHALLENGE QUESTIONS:

Can you change this base 2 number to base 8 using learning from above?

101101110001_2 = 5561 $_8$

5 5 6 1

Can you change this base 4 number to base 2 using learning from above?

3021_4 = 11001001 $_2$

11 00 10 01

RGB to HEX

RGB to HEX Conversion Worksheet:

Convert the following Hex values to RGB Decimal Values:

1. AC0BFF ________
2. 11AACB ________
3. FFCC00 ________
4. 660099 ________
5. FFFFFF ________ (What color is this?)
6. 000000 ________ (What color is this?)
7. BCABFF ________
8. 99FFA1 ________
9. AFBCA5 ________
10. 667AFB ________

Convert the following Decimal RGB values to Hex values

1. (255,100,87) ________
2. (109,0,124) ________
3. (86,10,91) ________
4. (255,255,255) ________ (What color is this?)
5. (0,0,0) ________ (What color is this?)
6. (102,101,77) ________
7. (10,18,122) ________
8. (134,29,37) ________
9. (57,123,48) ________
10. (255,18,73) ________

RGB to HEX Conversion Worksheet: KEY

Convert the following Hex values to RGB Decimal Values:

1. AC0BFF *172,11,255*
2. 11AACB *17, 170, 203*
3. FFCC00 *255,204,0*
4. 660099 *102,0,153*
5. FFFFFF *255,255,255* (What color is this?) *White*
6. 000000 *0,0,0* (What color is this?) *Black*
7. BCABFF *188,171,255*
8. 99FFA1 *153, 255, 161*
9. AFBCA5 *175,188,165*
10. 667AFB *102,122,251*

Convert the following Decimal RGB values to Hex values

1. (255,100,87) *FF6457*
2. (109,0,124) *6D007C*
3. (86,10,91) *560A5B*
4. (255,255,255) *FFFFFF* (What color is this?) *White*
5. (0,0,0) *000000* (What color is this?) *Black*
6. (102,101,77) *66654D*
7. (10,18,122) *0A127A*
8. (134,29,37) *861D25*
9. (57,123,48) *397B30*
10. (255,18,73) *FF1249*